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An extended TOPSIS model based on the Possibility theory under fuzzy environment

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ABSTRACT

This paper proposes an extended technique for order preference by similarity to ideal solution (TOPSIS) method under fuzzy environment to solve multi-attribute decision making (MADM) problem. The imprecise and fuzzy information of the MADM problem is expressed by triangular fuzzy number. The Possibility theory is used to handle triangular fuzzy numbers under fuzzy environment. In the extended TOPSIS method, the possibilistic mean value matrix and the possibilistic standard deviation matrix are constructed to compute the integrated relative closeness coefficient of each alternative. According to the integrated relative closeness coefficient, we can rank the preference order of all alternatives and select the most suitable one. Two numerical examples are presented to demonstrate the feasibility and efficiency of the new proposed TOPSIS method.

1. Introduction

Multi-attribute decision making (MADM) is devoted to solving the most desirable alternative selection problem according to multiple attributes. Recently MADM becomes a very important area of operational research and decision science. Especially, in the recent years, a lot of MADM methods have been proposed to solve MADM problems, such as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [14], AHP (Analytic Hierarchy Process) [22], DEMATEL (Decision Making Trial and Evaluation Laboratory) [12], VIKOR (*VIsekriterijumska optimizacija i KOmpromisno Resenje*) [21] and DEA (Data Envelopment Analytic) [7] methods. These MADM methods have been applied into solving lots of economics and management problems, such as supplier selection problem [26,23,27], virtual enterprise partner selection problem [33,32,34], plant layout design problem [31], evaluating sustainable transportation systems [4], partner choice problem in IS/IT outsourcing [9] and green supply chain management practices [20].

TOPSIS method, developed by Hwang and Yoon [14], is one of the well-known classical MADM methods. TOPSIS method, which refers to a technique for order preference by similarity to ideal solution, is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). The PIS is a set composing all the best values of each attribute, while the NIS is a set composing all the worst values of each attribute. TOPSIS method have received considerable attentions since it is proposed.

Traditionally, researchers dealt with complete and certain information environment and assumed that decision makers' evaluations on alternatives can be expressed by crisp values, then extended TOPSIS method to solve MADM problems under different conditions [18,1,25,2]. In many real practices, however, various types of uncertainties and imprecision often exist, the information of alternatives are vague, imprecise and uncertain by nature. Hence, researchers extended the TOPSIS method into fuzzy environment [34]. For example, Jahanshahloo et al. [16] developed a methodology for solving MADM problem with interval data by using the concept of TOPSIS. Shih et al. [24] proposed an extended TOPSIS method for group decision making where decision-makers' preferences were aggregated within the procedure. Chen and Tsao [10] proposed interval-valued fuzzy TOPSIS method based on interval-valued fuzzy data. Izadikhah [15] used Hamming distance to extend TOPSIS in a fuzzy environment. Vahdani et al. [28] developed a novel fuzzy modified TOPSIS method which could reflect both subjective judgment and objective information based on the concept of TOPSIS to solve MADM problem with multi-judges and multi-criteria under fuzzy environment. Beg and Rashid [5] extended fuzzy TOPSIS method for hesitant fuzzy linguistic term sets, and applied the extended fuzzy TOPSIS for the ranking of alternatives. Others extended TOPSIS method from different perspectives [17,3,13,33,6,32,19,30,29,34].

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In contrast to previous studies on the extensions for TOPSIS method under fuzzy environment, in this paper, we introduce the concept of Markowitz's portfolio mean–variance methodology into the traditional TOPSIS methods with fuzzy numbers, and propose a novel extended fuzzy TOPSIS method by utilizing the Possibility theory. As we know, when decision makers select alternative for their partners, most of them desire to select the ideal alternative with high expected return and low risk, so as to achieve a reasonable trade-off between return and risk. Thus, in the extended fuzzy TOPSIS method, the decision maker will select an ideal alternative with high possibilisic mean value and low possibilistic standard deviation, in which the possibilisic mean value is used as the measurement of investment return and the possibilistic standard deviation is viewed as the measurement of investment risk. And in the extended TOPSIS method the possibilistic mean values matrix and the possibilistic standard deviation are constructed to compute the integrated relative closeness coefficient of each alternative. The preference order of all alternatives can be ranked according to the relative closeness coefficient of each alternative. Moreover, we also compare the result of our new proposed method and existing methods through two examples.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts on the Possibility theory and TOPSIS method. Section 3 presents the new proposed fuzzy TOPSIS method based on Possibility theory. Section 4 presents two illustrative examples. Section 5 summarizes the paper.

2. Preliminaries

2.1. The Possibility theory

In this section, some basic concepts and definitions about Possibility theory are introduced. First, a fuzzy number A is a fuzzy set of the real line x with a normal, fuzzy convex and continuous membership function of bounded support [35,34,11]. The family of fuzzy numbers can be denoted by $F(x)$. $[A]^\gamma = \{t \in x | A(t) \geqslant \gamma\}$ if $\gamma > 0$ and $[A]^\gamma = \{t \in x | A(t) > 0\}$ if $\gamma = 0$ are defined as a γ-level set of fuzzy number A.

Definition 1. *Let $A \in x$ be fuzzy number with $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$, $\gamma \in [0,1]$. The possibilistic mean values of fuzzy number A is defined as $M(A) = \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma$, and the possibilistic variance of A as $Var(A) = \frac{1}{2}\int_0^1 \gamma(a_1(\gamma) - a_2(\gamma))^2 d\gamma$.*

Let A be a triangular fuzzy number with center a, left-width $\alpha > 0$, right -width $\beta > 0$, i.e., $A = (a - \alpha, a, a + \beta)$. Using Definition 1, a γ-level set of fuzzy number A can easily be computed as

$$[A]^\gamma = [a - (1-\gamma)\alpha,\ a + (1-\gamma)\beta], \quad \forall \gamma \in [0,1] \tag{1}$$

According to Definition 1, the possibilistic mean value of triangular fuzzy number A is described as

$$M(A) = \int_0^1 \gamma((a - (1-\gamma)\alpha) + (a + (1-\gamma)\beta))d\gamma = a + \frac{1}{6}(\beta - \alpha) \tag{2}$$

The possibilistic variance of triangular fuzzy number A can be written as

$$Var(A) = \frac{1}{2}\int_0^1 \gamma((a - (1-\gamma)\alpha) - (a + (1-\gamma)\beta))^2 d\gamma = \frac{1}{24}(\beta + \alpha)^2 \tag{3}$$

And the possibilistic standard deviation can be expressed as $\sqrt{Var(A)}$.

2.2. The TOPSIS method

The TOPSIS method procedure consists of the following steps:

(1) Design a set of attributes $C = \{C_j | \ j = 1, \ldots, m\}$.
(2) Generate a set of possible alternatives $X = \{X_i | \ i = 1, \ldots, n\}$.
(3) Construct the decision matrix $L = [a_{ij}]_{n \times m}$, where a_{ij} is the rating of alternative X_i with respect to attribute C_j.
(4) Decision maker elicits the weight for attribute C_j as ϖ_j, where $0 < \varpi_j < 1, \ j = 1, 2, \ldots, m$ and $\sum_{j=1}^m \varpi_j = 1$.
(5) Normalize the decision matrix. This step tries to transform various attribute dimensions into the non-dimensional attributes, which allows comparison across the attributes. The normalized value a'_{ij} is computed as

$$\begin{cases} a'_{ij} = \frac{a_{ij}}{a_j^+}, & j \in \Theta_1 \\ a'_{ij} = \frac{a_j^-}{a_{ij}}, & j \in \Theta_2 \end{cases} \tag{4}$$

where $a_j^+ = \max_i a_{ij}$, if $j \in \Theta_1$; $a_j^- = \min_i a_{ij}$, if $j \in \Theta_2$. Θ_1 is associated with benefit attribute, while Θ_2 is associated with cost attribute. And the normalized matrix can be written as $L' = \left[a'_{ij}\right]_{n \times m}$.

(6) Determine the PIS and the NIS as

$$E^+ = \{e_1^+, e_2^+, \ldots, e_m^+\}, \quad e_j^+ = \max_i a'_{ij} \tag{5}$$

$$E^- = \{e_1^-, e_2^-, \ldots, e_m^-\}, \quad e_j^- = \min_i a'_{ij} \tag{6}$$

(7) Compute the separation measure between each alternative and PIS as

$$d_i^+ = \left\{\sum_{j=1}^m \left(\left(a'_{ij} - e_j^+\right)w_j\right)^2\right\}^{\frac{1}{2}}, \quad 1, 2, \ldots, n \tag{7}$$

(8) Compute the separation measure between each alternative and NIS as

$$d_i^- = \left\{\sum_{j=1}^m \left(\left(a'_{ij} - e_j^-\right)w_j\right)^2\right\}^{\frac{1}{2}}, \quad i = 1, 2, \ldots, n \tag{8}$$

(9) Calculate the closeness coefficient of alternative X_i

$$U_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, 2, \ldots, n \tag{9}$$

(10) Rank all alternatives according to the closeness coefficient, select the best one.

3. The proposed fuzzy TOPSIS based on Possibility theory

In real world situation, it is hard for decision makers to get exact information about all alternatives. Therefore, decision makers often utilize fuzzy theory to evaluate alternatives. In this section, we propose a new and novel fuzzy TOPSIS method based on Possibility theory to support MADM process. First, we assume $X' = \{X'_i | \ i = 1, \ldots, n\}$ as a finite set of possible alternatives, $C' = \left\{C'_j | \ j = 1, \ldots, m\right\}$ as a finite set of attributes according to the desirability judgment of alternatives. Since the information of alternatives is fuzzy and uncertain during decision making process, the decision maker utilizes a triangular fuzzy number $\widetilde{l}_{ij}$ to estimate the judgment on alternative X'_i with respect to attribute C'_j, $\widetilde{l}_{ij} = (a_{ij} - \alpha_{ij}, a_{ij}, a_{ij} + \beta_{ij})$. Let $\widetilde{L} = [\widetilde{l}_{ij}]_{n \times m}$ be the decision matrix in the form of triangular fuzzy numbers. The MADM problem with triangular fuzzy numbers can be expressed in matrix format:

$$\widetilde{L}=\begin{bmatrix}(a_{11}-\alpha_{11},a_{11},a_{11}+\beta_{11}) & (a_{12}-\alpha_{12},a_{12},a_{12}+\beta_{12}) & \cdots & (a_{1m}-\alpha_{1m},a_{1m},a_{1m}+\beta_{1m})\\ (a_{21}-\alpha_{21},a_{21},a_{21}+\beta_{21}) & (a_{22}-\alpha_{22},a_{22},a_{22}+\beta_{22}) & \cdots & (a_{2m}-\alpha_{2m},a_{2m},a_{2m}+\beta_{2m})\\ \vdots & \vdots & \ddots & \vdots\\ (a_{n1}-\alpha_{n1},a_{n1},a_{n1}+\beta_{n1}) & (a_{n2}-\alpha_{n2},a_{n2},a_{n2}+\beta_{n2}) & \cdots & (a_{nm}-\alpha_{nm},a_{nm},a_{nm}+\beta_{nm})\end{bmatrix} \tag{10}$$

The decision maker gives weight w_j for attribute C_j' of each alternative, where $j=1,2,\ldots,m$. $w_j(j=1,2,\ldots,m)$ belongs to $[0,1]$ and sums up to one.

$$0\leqslant w_j\leqslant 1,\quad j=1,2,\ldots,m,\quad \sum_{j=1}^{m} w_j=1 \tag{11}$$

The values of different attributes have different dimensions. Thus, the fuzzy decision matrix $\widetilde{L}=[\widetilde{l_{ij}}]_{n\times m}$ should be transformed into normalized fuzzy matrix in order to reduce disturbance in the final results. Let $\widetilde{L}'=[\widetilde{l'_{ij}}]_{n\times m}$ be the normalized fuzzy matrix in the form of triangular fuzzy numbers, where $\widetilde{l'_{ij}}=\left(a'_{ij}-\alpha'_{ij},a'_{ij},a'_{ij}+\beta'_{ij}\right)$.

In general, there are two attribute categories for alternatives: benefit type and cost type. The higher the benefit type value is, the better it will be. It is opposite for the cost type. To avoid the complicated normalization formula used in classical TOPSIS method, the linear scale transformation method proposed by Chen [8] is used to transform different attribute scales into a comparable scale.

$$\widetilde{l'_{ij}}=\left(\frac{a_{ij}-\alpha_{ij}}{(a_{ij}+\beta_{ij})^+},\frac{a_{ij}}{(a_{ij}+\beta_{ij})^+},\frac{a_{ij}+\beta_{ij}}{(a_{ij}+\beta_{ij})^+}\right),\quad j\in\Theta_1 \tag{12}$$

$$\widetilde{l'_{ij}}=\left(\frac{(a_{ij}-\beta_{ij})^-}{a_{ij}+\beta_{ij}},\frac{(a_{ij}-\beta_{ij})^-}{a_{ij}},\frac{(a_{ij}-\beta_{ij})^-}{a_{ij}-\alpha_{ij}}\right),\quad j\in\Theta_2 \tag{13}$$

where $(a_{ij}+\beta_{ij})^+=\max_i(a_{ij}+\beta_{ij})$, if $j\in\Theta_1$; $(a_{ij}-\beta_{ij})^-=\min_i(a_{ij}-\beta_{ij})$, if $j\in\Theta_2$. Θ_1 is associated with benefit attribute, while Θ_2 is associated with cost attribute.

The normalization approach mentioned above is to make the ranges of normalized triangular fuzzy numbers belong to $[0,1]$.

According to formula (2), we can get the possibilistic mean value of triangular fuzzy number $\widetilde{l'_{ij}}=\left(a'_{ij}-\alpha'_{ij},a'_{ij},a'_{ij}+\beta'_{ij}\right)$ as follows:

$$M\left(\widetilde{l'_{ij}}\right)=a'_{ij}+\frac{1}{6}\left(\beta'_{ij}-\alpha'_{ij}\right) \tag{14}$$

And according to formula (3), the possibilistic variance of triangular fuzzy number $\widetilde{l'_{ij}}=\left(a'_{ij}-\alpha'_{ij},a'_{ij},a'_{ij}+\beta'_{ij}\right)$ can be written as

$$Var\left(\widetilde{l'_{ij}}\right)=\frac{1}{24}\left(\beta'_{ij}+\alpha'_{ij}\right)^2 \tag{15}$$

According to formula (15), we obtain the standard deviation about the triangular fuzzy number $\widetilde{l'_{ij}}=\left(a'_{ij}-\alpha'_{ij},a'_{ij},a'_{ij}+\beta'_{ij}\right)$ as

$$StD\left(\widetilde{l'_{ij}}\right)=\sqrt{Var\left(\widetilde{l'_{ij}}\right)}=\sqrt{\frac{1}{24}}\left(\beta'_{ij}+\alpha'_{ij}\right) \tag{16}$$

Therefore, the possibilistic mean value matrix $M(\widetilde{L})=\left[M\left(\widetilde{l'_{ij}}\right)\right]_{n\times m}$ about the normalized fuzzy matrix $\widetilde{L}'=\left[\widetilde{l'_{ij}}\right]_{n\times m}$ can be described as

$$M(\widetilde{L})=\begin{bmatrix}M(\widetilde{l'_{11}}) & M(\widetilde{l'_{12}}) & \cdots & M(\widetilde{l'_{1m}})\\ M(\widetilde{l'_{21}}) & M(\widetilde{l'_{22}}) & \cdots & M(\widetilde{l'_{2m}})\\ \vdots & \vdots & \ddots & \vdots\\ M(\widetilde{l'_{n1}}) & M(\widetilde{l'_{n2}}) & \cdots & M(\widetilde{l'_{nm}})\end{bmatrix} \tag{17}$$

And the possibilistic standard deviation matrix $StD(\widetilde{L})=[StD(\widetilde{l'_{ij}})]_{n\times m}$ about the normalized fuzzy matrix $\widetilde{L}'=[\widetilde{l'_{ij}}]_{n\times m}$ can be written as

$$StD(\widetilde{L})=\begin{bmatrix}StD(\widetilde{l'_{11}}) & StD(\widetilde{l'_{12}}) & \cdots & StD(\widetilde{l'_{1m}})\\ StD(\widetilde{l'_{21}}) & StD(\widetilde{l'_{22}}) & \cdots & StD(\widetilde{l'_{2m}})\\ \vdots & \vdots & \ddots & \vdots\\ StD(\widetilde{l'_{n1}}) & StD(\widetilde{l'_{n2}}) & \cdots & StD(\widetilde{l'_{nm}})\end{bmatrix} \tag{18}$$

Based on the TOPSIS concept, we identify PIS $M(\widetilde{L})^+$ and NIS $M(\widetilde{L})^-$ about the possibilistic mean value matrix $M(\widetilde{L})$ for the decision maker as:

$$M(\widetilde{L})^+=(M(\widetilde{l_1})^+,M(\widetilde{l_2})^+,\ldots,M(\widetilde{l_m})^+) \tag{19}$$

$$M(\widetilde{L})^-=(M(\widetilde{l_1})^-,M(\widetilde{l_2})^-,\ldots,M(\widetilde{l_m})^-) \tag{20}$$

where $M(\widetilde{l_j})^+=\max_i M(\widetilde{l_{ij}}),M(\widetilde{l_j})^-=\min_i M(\widetilde{l_{ij}}),i=1,2,\ldots,n.$

Moreover, we also define PIS $StD(\widetilde{L})^+$ and NIS $StD(\widetilde{L})^-$ about the possibilistic standard deviation matrix $StD(\widetilde{L})=[StD(\widetilde{l'_{ij}})]_{n\times m}$ for the decision maker as:

$$StD(\widetilde{L})^+=(StD(\widetilde{l_1})^+,StD(\widetilde{l_2})^+,\ldots,StD(\widetilde{l_m})^+) \tag{21}$$

$$StD(\widetilde{L})^-=(StD(\widetilde{l_1})^-,StD(\widetilde{l_2})^-,\ldots,StD(\widetilde{l_m})^-) \tag{22}$$

where $StD(\widetilde{l_j})^+=\min_i StD(\widetilde{l_{ij}}),StD(\widetilde{l_j})^-=\max_i StD(\widetilde{l_{ij}}),\ i=1,2,\ldots,n.$

By using the m-dimensional Euclidean distance, the separation measures of each alternative's possibilistic mean value from the PIS $M(\widetilde{L})^+$ and possibilistic standard deviation from the PIS $StD(\widetilde{L})^+$ are given as, respectively:

$$d_i(M(\widetilde{L})^+)=\sqrt{\sum_{j=1}^{m}((M(\widetilde{l_j})^+-M(\widetilde{l_{ij}}))\varpi_j)^2},\quad i=1,2,\ldots,n \tag{23}$$

$$d_i(StD(\widetilde{L})^+)=\sqrt{\sum_{j=1}^{m}((StD(\widetilde{l_j})^+-StD(\widetilde{l_{ij}}))\varpi_j)^2},\quad i=1,2,\ldots,n \tag{24}$$

Similarly, the separation measures of each alternative's possibilistic mean value from the NIS $M(\widetilde{L})^-$ and possibilistic standard deviation from the NIS $StD(\widetilde{L})^-$ are given as, respectively:

$$d_i(M(\widetilde{L})^-)=\sqrt{\sum_{j=1}^{m}((M(\widetilde{l_j})^--M(\widetilde{l_{ij}}))\varpi_j)^2},\quad i=1,2,\ldots,n \tag{25}$$

$$d_i(StD(\widetilde{L})^-)=\sqrt{\sum_{j=1}^{m}((StD(\widetilde{l_j})^--StD(\widetilde{l_{ij}}))\varpi_j)^2},\quad i=1,2,\ldots,n \tag{26}$$

A closeness coefficient is defined to determine the ranking order of all alternatives once $d_i(M(\widetilde{L})^+)$, $d_i(StD(\widetilde{L})^+)$ and $d_i(M(\widetilde{L})^-)$, $d_i(StD(\widetilde{L})^-)$ of each alternative X_i' are calculated. The relative closeness coefficient of alternative X_i' about its possibilistic mean value and standard deviation are defined as

$$\mu_i(M(\widetilde{L}))=\frac{d_i(M(\widetilde{L})^-)}{d_i(M(\widetilde{L})^-)+d_i(M(\widetilde{L})^+)},\quad i=1,2,\ldots,n \tag{27}$$

$$\mu_i(StD(\widetilde{L}))=\frac{d_i(StD(\widetilde{L})^-)}{d_i(StD(\widetilde{L})^-)+d_i(StD(\widetilde{L})^+)},\quad i=1,2,\ldots,n \tag{28}$$

Then the integrated relative closeness coefficient of alternative X_i' is given as

$$\mu_i=\sqrt{StD(\widetilde{L})\times M(\widetilde{L})},\quad i=1,2,\ldots,n \tag{29}$$

Hence, according to the integrated relative closeness coefficient, the ranking order of all alternatives can be determined, and the decision maker selects the best one among a set of feasible alternatives.

As a summary, the procedure to find out the best alternative with the extended fuzzy TOPSIS method based on Possibility theory is shown in the following.

Step1: Generate a set of possible alternatives $X' = \{X'_i|\ i = 1, \ldots, n\}$ and design a set of attributes $C' = \left\{C'_j|\ j = 1, \ldots, m\right\}$.

Step2: Construct the fuzzy decision matrix $\widetilde{L} = [\widetilde{l}_{ij}]_{n\times m}$ with triangular fuzzy numbers, and transform the fuzzy decision matrix $\widetilde{L} = [\widetilde{l}_{ij}]_{n\times m}$ into the normalized fuzzy decision matrix $\widetilde{L}' = [\widetilde{l'_{ij}}]_{n\times m}$. This step tries to transform various attribute dimensions into the non-dimensional attributes. Formulas (12) and (13) are used for computing the normalized triangular fuzzy numbers.

Step3: Construct the possibilistic mean value matrix $M(\widetilde{L}) = [M(\widetilde{l'_{ij}})]_{n\times m}$ by using formula (14) and the possibilistic standard deviation matrix $StD(\widetilde{L}) = [StD(\widetilde{l'_{ij}})]_{n\times m}$ by using formula (16).

Step4: Decision maker elicits weight for attribute C'_j as ϖ_j, where $j = 1, 2, \ldots, m$.

Step5: Identify PIS $M(\widetilde{L})^+$ and NIS $M(\widetilde{L})^-$ about the possibilistic mean value matrix $M(\widetilde{L})$ by using formulas (19) and (20).

Step6: Define PIS $StD(\widetilde{L})^+$ and NIS $StD(\widetilde{L})^-$ about the possibilistic standard deviation matrix $StD(\widetilde{L})$ by using formulas (21) and (22).

Step7: Compute the separation measure of each alternative's possibilistic mean value from the PIS $M(\widetilde{L})^+$ and possibilistic standard deviation from the PIS $StD(\widetilde{L})^+$ by using formulas (23) and (24).

Step8: Compute the separation measure each alternative's possibilistic mean value from the NIS $M(\widetilde{L})^-$ and possibilistic standard from the NIS $StD(\widetilde{L})^-$ by using formulas (25) and (26).

Step9: Calculate closeness coefficient of alternative X'_i about its possibilistic mean value and possibilistic standard deviation by using formulas (27) and (28), and the integrated relative closeness coefficient of alternative X'_i by using formula (29).

Step10: Rank the preference order of all alternatives according to the integrated relative closeness coefficient, and select the best one.

4. Numerical examples

In this section, we use two examples on MADM problem with triangular fuzzy numbers to illustrate the proposed extended TOPSIS approach in this paper.

Example 1. The virtual enterprise (VE) is of increasing importance due to its flexibility, agility and efficiency. A major issue in the formation of a VE is to choose suitable partners during the establishing process [32]. Here, suppose a core enterprise wants to select the best partner from four alternatives (X'_1, X'_2, X'_3 and X'_4) to form a new VE and realize the emerging market opportunity. The partner selection decision is based on five attributes, including cost (C'_1), relationship closeness (C'_2), completion probability (C'_3), time (C'_4) and quality (C'_5). C'_1 and C'_4 are cost type attributes, while C'_2, C'_3 and C'_5 are benefit type attributes.

The decision matrix with triangular fuzzy numbers and weights of five attributes is shown in Table 1. The proposed approach is currently applied to solve the MADM problem and the computational procedure is given as follows:

Table 1
The fuzzy decision matrix and weights of five attributes for Example 1.

Alternative	Alternative				
	C'_1	C'_2	C'_3	C'_4	C'_5
X'_1	[11,1,1]	[81,1,3]	[0.96,0.1,0.2]	[21,1,4]	[0.95,0.1,0.2]
X'_2	[12,1,3]	[85,1,1]	[0.94,0.2,0.2]	[24,0,1]	[0.96,0.1,0.0]
X'_3	[13,2,1]	[87,2,2]	[0.88,0.1,0.4]	[22,1,1]	[0.94,0.2,0.3]
X'_4	[13,1,3]	[92,1,2]	[0.89,0.2,0.2]	[21,2,2]	[0.95,0.1,0.3]
Weight	0.22	0.17	0.25	0.15	0.21

Step 1: : Use triangular fuzzy numbers to evaluate alternatives with respect to each attribute, as shown in Table 1.

Step 2: : Normalizes the fuzzy decision matrix by using formulas (12) and (13), as shown in Table 2.

Step 3: Construct the possibilistic mean value matrix by using formula (14) and possibilistic standard variance matrix by using formula (16):

$$M(\widetilde{L}) = \begin{bmatrix} 0.912 & 0.865 & 0.981 & 0.888 & 0.971 \\ 0.818 & 0.904 & 0.959 & 0.786 & 0.978 \\ 0.783 & 0.926 & 0.903 & 0.864 & 0.961 \\ 0.756 & 0.980 & 0.908 & 0.908 & 0.973 \end{bmatrix} \tag{30}$$

$$StD(\widetilde{L}) = \begin{bmatrix} 0.0340 & 0.0087 & 0.0063 & 0.0388 & 0.0063 \\ 0.0497 & 0.0043 & 0.0083 & 0.0065 & 0.0021 \\ 0.0398 & 0.0087 & 0.0104 & 0.0161 & 0.0104 \\ 0.0425 & 0.0065 & 0.0083 & 0.0355 & 0.0083 \end{bmatrix} \tag{31}$$

Step 4: Determine PIS and NIS about the possibilistic mean value matrix as follows:

$$M(\widetilde{L})^+ = (0.912, 0.980, 0.981, 0.908, 0.978) \tag{32}$$

$$M(\widetilde{L})^- = (0.756, 0.865, 0.903, 0.786, 0.961) \tag{33}$$

Step 5: Determine PIS and NIS about possibilistic standard deviation matrix as follows:

$$StD(\widetilde{L})^+ = (0.0340, 0.0043, 0.0063, 0.0065, 0.0021) \tag{34}$$

$$StD(\widetilde{L})^- = (0.0497, 0.0087, 0.0104, 0.0388, 0.0104) \tag{35}$$

Step6: Calculate the separation measure about each alternative's possibilistic mean value from PIS and NIS, as shown in Table 3.

Step 7: : Compute the separation measure about each alternative's possibilistic standard deviation from PIS and NIS, as shown in Table 4.

Step 8: Calculate the closeness coefficients of each alternative as shown in Table 5.

From Table 5, we can find that the ranking order of four alternatives is $X'_1 \succ X'_4 \succ X'_2 \succ X'_3$ according to $\mu_i(M(\widetilde{L}))$, while the ranking order is $X'_2 \succ X'_3 \succ X'_1 \succ X'_4$ according to $\mu_i(StD(\widetilde{L}))$. Hence, by using formula (29), we can get the ranking order according to the integrated relative closeness coefficient is $X'_1 \succ X'_2 \succ X'_3 \succ X'_4$. Thus, the finaloptimal" alternative is X'_1.

In addition, we use the extended TOPSIS method proposed by Chen [8] to solve this partner selection problem of VE, with the yielded result shown in Fig. 1. From Fig. 1 we can see, the ranking order for alternatives yielded by Chen [8] is $X'_1 \succ X'_4 \succ X'_3 \succ X'_2$. Though the most desirable alternative is still X'_1, but the ranking

Table 2
The normalized fuzzy decision matrix for Example 1.

Alternative	Alternative				
	C'_1	C'_2	C'_3	C'_4	C'_5
X'_1	[0.83, 0.91, 1.00]	[0.85, 0.86, 0.89]	[0.97, 0.98, 1.00]	[0.76, 0.90, 0.95]	[0.96, 0.97, 0.99]
X'_2	[0.67, 0.83, 0.91]	[0.89, 0.90, 0.91]	[0.94, 0.96, 0.98]	[0.76, 0.79, 0.79]	[0.97, 0.98, 0.98]
X'_3	[0.71, 0.77, 0.91]	[0.90, 0.93, 0.95]	[0.89, 0.90, 0.94]	[0.83, 0.86, 0.90]	[0.94, 0.96, 0.99]
X'_4	[0.63, 0.77, 0.83]	[0.97, 0.98, 1.00]	[0.89, 0.91, 0.93]	[0.83, 0.90, 1.00]	[0.96, 0.97, 1.00]

Table 3
The separation measure about each alternative's possibilistic mean value for Example 1.

Separation measure	Alternative			
	X'_1	X'_2	X'_3	X'_4
$d_i(M(\widetilde{L})^+)$	0.0266	0.0328	0.0308	0.0344
$d_i(M(\widetilde{L})^-)$	0.0424	0.0210	0.0167	0.0269

Table 4
The separation measure about each alternative's possibilistic standard deviation for Example 1.

Separation measure	Alternative			
	X'_1	X'_2	X'_3	X'_4
$d_i(StD(\widetilde{L})^+)$	0.0050	0.0035	0.0029	0.0050
$d_i(StD(\widetilde{L})^-)$	0.0037	0.0052	0.0041	0.0018

Table 5
The Closeness coefficient and ranking of each alternative for Example 1.

Alternative	Closeness coefficient			
	$\mu_i(M(\widetilde{L}))$	$\mu_i(StD(\widetilde{L}))$	μ_i	Ranking
X'_1	0.6140	0.4268	0.5119	1
X'_2	0.3909	0.6003	0.4844	2
X'_3	0.3524	0.5840	0.4536	3
X'_4	0.4394	0.2692	0.3439	4

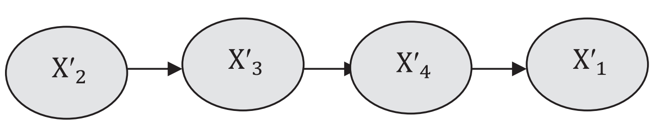

Fig. 1. The ranking order of alternatives obtained by Chen [8].

order is quite different from our result, due to the fact that we take both the return and risk into consideration and use the integrated relative closeness coefficient as the ranking criterion.

Example 2. Here we demonstrate how to apply the proposed TOPSIS method for supplier selection problem. Assume a manufacturing firm wants to select the most suitable supplier from candidates (X''_1, X''_2, X''_3 and X''_4). The manufacturing firm determines five attributes to evaluate candidates: (1) supplier's profile (C''_1); (2) overall cost of the material (C''_2); (3) delivery capability (C''_3); (4) quality of the material (C'_4); and (5) relationship closeness (C''_5). Clearly, C''_2 is cost type attribute and the others are benefit type attributes. Assume that the assessments of candidates according to the five attributes are provided as triangular fuzzy numbers, which are shown in Table 6.

Table 6
The triangular fuzzy decision matrix for four alternatives for Example 2.

Alternative	Alternative				
	C''_1	C''_2	C''_3	C''_4	C''_5
X''_1	[9, 1, 1]	[8, 1, 3]	[96, 4, 3]	[0.94, 0.2, 0.2]	[95, 1, 2]
X''_2	[8, 2, 1]	[9, 1, 1]	[94, 2, 2]	[0.93, 0.1, 0.2]	[96, 1, 0]
X''_3	[8, 1, 2]	[9, 2, 1]	[88, 5, 4]	[0.94, 0.2, 0.3]	[95, 2, 3]
X''_4	[10, 1, 0]	[11, 1, 2]	[89, 2, 2]	[0.96, 0.1, 0.3]	[95, 1, 3]

Furthermore, we assume that the manufacturing firm determines the weight vector about the five attributes as the following: $W = (0.15, 0.15, 0.25, 0.30, 0.15)^T$.

In order to apply the extended TOPSIS method, the fuzzy decision matrix in Table 6 needs to be normalized. The normalized triangular fuzzy decision matrix by using formulas (12) and (13) is shown as:

$$\widetilde{L}' = \begin{bmatrix} (0.80, 0.90, 1.00) & (0.64, 0.88, 1.00) & (0.93, 0.97, 1.00) & (0.93, 0.95, 0.97) & (0.96, 0.97, 0.99) \\ (0.60, 0.80, 0.90) & (0.70, 0.78, 0.88) & (0.93, 0.95, 0.97) & (0.93, 0.94, 0.96) & (0.97, 0.98, 0.98) \\ (0.70, 0.80, 1.00) & (0.70, 0.78, 1.00) & (0.84, 0.89, 0.93) & (0.93, 0.95, 0.98) & (0.95, 0.97, 1.00) \\ (0.90, 1.00, 1.00) & (0.54, 0.64, 0.70) & (0.88, 0.90, 0.92) & (0.96, 0.97, 1.00) & (0.96, 0.97, 1.00) \end{bmatrix} \tag{36}$$

In the light of formula (36), we can construct the possibilistic mean value matrix and the possibilistic standard variance matrix as follows, respectively:

$$M(\widetilde{L}) = \begin{bmatrix} 0.900 & 0.856 & 0.968 & 0.949 & 0.971 \\ 0.783 & 0.781 & 0.949 & 0.941 & 0.978 \\ 0.817 & 0.802 & 0.887 & 0.951 & 0.971 \\ 0.983 & 0.631 & 0.899 & 0.973 & 0.973 \end{bmatrix} \tag{37}$$

$$StD(\widetilde{L}) = \begin{bmatrix} 0.0408 & 0.0742 & 0.0144 & 0.0083 & 0.0063 \\ 0.0612 & 0.0357 & 0.0083 & 0.0062 & 0.0021 \\ 0.0612 & 0.0612 & 0.0186 & 0.0103 & 0.0104 \\ 0.0204 & 0.0330 & 0.0083 & 0.0083 & 0.0083 \end{bmatrix} \tag{38}$$

Next, we can determine PIS and NIS about the possibilistic mean value matrix and possibilistic standard deviation matrix as follows:

$$M(\widetilde{L})^+ = (0.983, 0.856, 0.968, 0.973, 0.978) \tag{39}$$

$$M(\widetilde{L})^- = (0.783, 0.631, 0.887, 0.941, 0.971) \tag{40}$$

$$StD(\widetilde{L})^+ = (0.0204, 0.0330, 0.0083, 0.0062, 0.0021) \tag{41}$$

$$StD(\widetilde{L})^- = (0.0612, 0.0742, 0.0186, 0.0103, 0.0104) \tag{42}$$

Thus, the separation measure about each alternative's possibilistic mean value from PIS and NIS can be calculated, respectively, as shown in Table 7.

And the separation measure about each alternative's possibilistic standard deviation from PIS and NIS are computed in Table 8.

In the end, we calculate the closeness coefficient of each alternative in Table 9. In the second column of Table 9, calculated

Table 7
The separation measure about each alternative's possibilistic mean value for Example 2.

Separation measure	Alternative			
	X''_1	X''_2	X''_3	X''_4
$d_i(M(\widetilde{L})^+)$	0.0144	0.0337	0.0338	0.0380
$d_i(M(\widetilde{L})^-)$	0.0432	0.0274	0.0263	0.0317

Table 8
The separation measure about each alternative's possibilistic standard deviation for Example 2.

Separation measure	Alternative			
	X''_1	X''_2	X''_3	X''_4
$d_i(StD(\widetilde{L})^+)$	0.0071	0.0061	0.0081	0.0011
$d_i(StD(\widetilde{L})^-)$	0.0034	0.0066	0.0019	0.0091

Table 9
The closeness coefficient and ranking of each alternative for Example 2.

Alternative	Closeness coefficient			
	$\mu_i(M(\widetilde{L}))$	$\mu_i(StD(\widetilde{L}))$	μ_i	Ranking
X''_1	0.7506	0.3196	0.4898	2
X''_2	0.4485	0.5173	0.4814	3
X''_3	0.4378	0.1942	0.2916	4
X''_4	0.4550	0.8906	0.6366	1

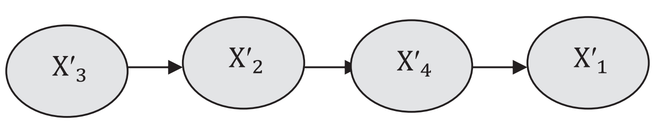

Fig. 2. The ranking order of alternatives obtained by Jahanshahloo et al. [16].

closeness coefficients for all alternatives about their possibilistic mean values are given. According to $\mu_i(M(\widetilde{L}))$, the ranking order of alternatives is $X''_1 \succ X''_4 \succ X''_2 \succ X''_3$. Therefore, the best candidate is X''_1 in term of $\mu_i(M(\widetilde{L}))$. However, according to $\mu_i(StD(\widetilde{L}))$, the ranking order of alternatives is $X''_4 \succ X''_2 \succ X''_1 \succ X''_3$, as shown in the third column of Table 9. This means that candidate X''_4 is the optimal supplier in terms of $\mu_i(StD(\widetilde{L}))$. Then we calculate the integrated closeness coefficients of four alternatives, as shown in the fourth column of Table 9. In accordance with the integrated relative closeness coefficient of each alternative, the order preference is $X''_4 \succ X''_1 \succ X''_2 \succ X''_3$, and thus the most desirable supplier is alternative X''_4.

Similarly, we compare our result with that of Jahanshahloo et al. [16], which is shown in Fig. 2. It is clearly that our result is quite different from that of Jahanshahloo et al. [16], in which X'_1 is the most desirable supplier. This is because that we assume that the manufacturing firm desires to select a supplier with high possibilistic mean value and low standard deviation, so as to obtain a reasonable trade-off between return and risk.

5. Conclusions

In order to deal with vague and incomplete information in MADM problem, some TOPSIS methods with fuzzy numbers are proposed in traditional literature. However, existing fuzzy TOPSIS methods did not consider the decision-making risk under fuzzy environment. In this paper we introduce the concept of Markowitz's portfolio mean–variance methodology into the traditional TOPSIS methods with fuzzy numbers, and propose a novel extended fuzzy TOPSIS method by utilizing the Possibility theory. In the extended fuzzy TOPSIS method, the decision maker will select an ideal alternative with high possibilisic mean value and low possibilistic standard deviation, in which the possibilisic mean value is used as the measurement of investment return and the possibilistic standard deviation is viewed as the measurement of investment risk. Two numerical examples are presented to illustrate the proposed extended TOPSIS method. The comparisons between the results of the new proposed method and other existing methods are offered. Of course, our extended TOPSIS method can not only be applied to partner selection and supplier selection problems, but also be used into other purposes, such as evaluating performance, information systems evaluation, and human resource arrangements. In addition, since adventurous or conservative decision makers would make quite different decisions under the same circumstance, thus, considering decision makers' different risk preferences in the proposed TOPSIS model would be an interesting point to be investigated in the further.

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